Sums and differences of angles

1. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
2. $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
3. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
4. $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$
5. $\tan(a+b) = \dfrac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$
6. $\tan(a-b) = \dfrac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$

Example 1

Expand and simplify $\cos(2a-b) - \cos(2a+b)$.

Using formula (4), $\cos(2a-b) = \cos(2a)\cos(b) + \sin(2a)\sin(b)$

Using formula (3), $\cos(2a+b) = \cos(2a)\cos(b) - \sin(2a)\sin(b)$

$\cos(2a-b) - \cos(2a+b) =$

$\cos(2a)\cos(b) + \sin(2a)\sin(b) - \cos(2a)\cos(b) + \sin(2a)\sin(b) = 2\sin(2a)\sin(b)$

$\therefore \cos(2a-b) - \cos(2a+b) = 2\sin(2a)\sin(b)$

Example 2

Without using a calculator, find the exact value of $\cos 105^\circ$.

Hint: Write 105° as a sum of two angles with known ratios.

$\cos 105^\circ = \cos(60^\circ + 45^\circ) =$

$$\cos(60^\circ)\cos(45^\circ) - \sin(60^\circ)\sin(45^\circ) = \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\therefore \cos 105^\circ = \frac{\sqrt{2}-\sqrt{6}}{4}$$