

## Absolute Value

### Definition

$$|x| = a \text{ means } x = \pm a$$

$$|x| < a \text{ means } -a < x < a$$

$$|x| > a \text{ means } x < -a \text{ and } x > a$$

A. Solve :

(a)  $|x + 2| = 7$  or  $x + 2 = \pm 7$

$$x + 2 = 7$$

$$x = 5$$

$$x + 2 = -7$$

$$x = -9$$

$$\therefore x = 5, x = -9$$

(b)  $|2a| > 4$  or  $2a < -4$  and  $2a > 4$

$$\therefore a < -2 \text{ and } a > 2$$

(c)  $|x - 5| \leq 1$

$$-1 \leq x - 5 \leq 1$$

$$-1 + 5 \leq x - 5 + 5 \leq 1 + 5$$

$$\therefore 4 \leq x \leq 6$$

B. Solve :

$$|x + 2| = 5x - 3$$

Case 1 :  $x + 2 = 5x - 3$

$$x - 5x = -3 - 2$$

$$-4x = -5$$

$$x = \frac{5}{4} = 1\frac{1}{4}$$

Check by substitution :

$$\left| 1\frac{1}{4} + 2 \right| = 5 \times 1\frac{1}{4} - 3$$

$$3\frac{1}{4} = 3\frac{1}{4} \text{ True}$$

$$\therefore x = 1\frac{1}{4} \text{ is a solution of } |x + 2| = 5x - 3$$

Case 2 :  $-(x + 2) = 5x - 3$

$$-x - 2 = 5x - 3$$

$$-x - 5x = -3 + 2$$

$$-6x = -1$$

Check by substitution :

$$\left| \frac{1}{6} + 2 \right| = 5 \times \frac{1}{6} - 3$$

$$2\frac{1}{6} = -2\frac{1}{6} \text{ False}$$

$$\therefore x = \frac{1}{6} \text{ is not a solution of } |x + 2| = 5x - 3$$

Answer :  $\therefore x = 1\frac{1}{4}$

C. Solve :

$$(a) |x + 3| = |3x - 1|$$

$$\text{Case 1: } x + 3 = 3x - 1$$

$$x - 3x = -1 - 3$$

$$-2x = -4$$

$$\therefore x = 2$$

Check by substitution :

$$|2 + 3| = |3 \times 2 - 1|$$

$$5 = 5 \text{ True}$$

$x = 2$  is a solution of  $|x + 3| = |3x - 1|$

$$\text{Case 2: } x + 3 = -(3x - 1)$$

$$x + 3 = -3x + 1$$

$$x + 3x = 1 - 3$$

$$4x = -2$$

$$\therefore x = -\frac{1}{2}$$

Check by substitution :

$$\left| -\frac{1}{2} + 3 \right| = \left| 3 \times \left( -\frac{1}{2} \right) - 1 \right|$$

$$\left| 2\frac{1}{2} \right| = \left| -2\frac{1}{2} \right|$$

$$2\frac{1}{2} = 2\frac{1}{2} \text{ True}$$

$x = -\frac{1}{2}$  is a solution of  $|x + 3| = |3x - 1|$

Case 3 :

$$-(x + 3) = 3x - 1$$

$$-x - 3 = 3x - 1$$

$$-x - 3x = -1 + 3$$

$$-4x = 2$$

$$\therefore x = -\frac{1}{2}$$

Case 4 :

$$-(x + 3) = -(3x - 1)$$

$$x + 3 = 3x - 1$$

$$x - 3x = -1 - 3$$

$$-2x = -4$$

$$\therefore x = 2$$

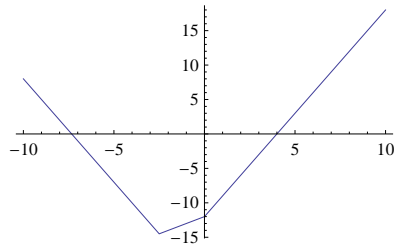
Answer :  $x = 2, x = -\frac{1}{2}$

The next questions are solved using *Mathematica* software and the working out is similar to the previous questions.

**A. Solve** `Abs[2 x + 5] + Abs[x] == 17, x]`

$$\left\{ \left\{ x \rightarrow -\frac{22}{3} \right\}, \left\{ x \rightarrow 4 \right\} \right\}$$

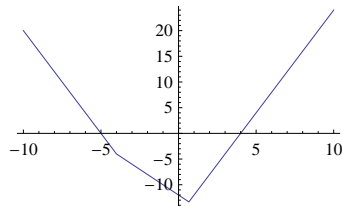
`Plot[Abs[2 x + 5] + Abs[x] - 17 == 0, {x, -10, 10}]`



`B. Solve[Abs[3 d - 2] + Abs[d + 4] == 18, d]`

`{{d -> -5}, {d -> 4}}`

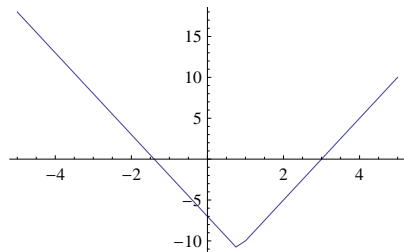
`Plot[Abs[3 d - 2] + Abs[d + 4] - 18 == 0, {d, -10, 10}]`



`C. Solve[Abs[4 t - 3] + Abs[t - 1] == 11, t]`

`{{t -> -7/5}, {t -> 3}}`

`Plot[Abs[4 t - 3] + Abs[t - 1] - 11 == 0, {t, -5, 5}]`



`D. Reduce[Abs[4 t - 3] + Abs[t - 1] < 11, t]`

`$-\frac{7}{5} < t < 3$`